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Geometric programming approach in multivariate stratified sample surveys in case of non-response

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This paper provides an attempt to utilize the geometric programming approach in multivariate stratified sample surveys in case of non-response. The problem has been solved in two phases. In first phase the multivariate stratified sample surveys in case of non-response has been formulated as geometric programming problem (GPP) and the solution is obtained. The obtained solution is the dual solution of the formulated GPP. In second phase with the help of dual solutions of formulated GPP and primal-dual relationship theorem the optimum allocation of sample sizes of respondents and non-respondents are obtained. A numerical example is given to illustrate the procedure.

keywords: geometric programming, convex programming, non-response, optimum allocation, multivariate stratified sampling.

1 Introduction

In stratified sampling heterogeneous population is converted into a homogeneous population by dividing it into homogeneous stratum. The maximum precision will be obtained with the best choices of the sample sizes. Many authors have discussed multivariate stratified sample survey problems. Among them are Neyman (1934), Geary (1949), Dalenius (1957), Ghosh (1958), Yates (1960), Aoyama (1962), Folks and Antle (1965), Kokan and Khan (1967), Chatterjee (1968),Chatterjee (1972), Ahsan (1978), Ahsan (1975), Ahsan

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and Khan (1977), Khan et al. (1997), Chromy (1987), Jahan et al. (1994), Jahan et al. (2001), Jahan and Ahsan (1995), Singh (2003), Khan et al. (2003), Khan et al. (2010) and Khowaja et al. (2011). Recent work has been done on multivariate stratified sample survey problems using different techniques by Khan et al. (2012), Varshney et al. (2011), Ali et al. (2013).

In multivariate stratified sample survey problems, non-response appears when the required information cannot be obtained. The problem of non-response may occur due to the refusal by respondents or they are not at home making the information of sample inaccessible. The problem of non-response occurs in almost all surveys. The extent of non-response depends on various factors such as type of the target population, type of the survey and the time of survey. For dealing the problem of non-response the population is divided into two disjoint groups one of respondents and another of non-respondents. For the stratified sampling it may be assumed that every stratum is divided into two mutually exclusive and exhaustive groups of respondents and non-respondents.

Hansen and Hurwitz (1946) presented a classical non-response theory which was first developed for the surveys in which the first attempt was made by mailing the questionnaires and a second attempt was made by personal interview to a sub sample of the non-respondents. They constructed estimator for the population mean and derived expression for its variance and also worked out optimum sampling fraction among the non-respondents. El-Badry (1956) further extended the Hansen and Hurwitz's technique by sending waves of questionnaires to the non-respondent units to increase the response rate. The generalized El-Badry's approach for different sampling design was given by Foradari (1961). Srinath (1951) suggested the selection of sub samples by making several attempts. Khare (1987) investigated the problem of optimum allocation in stratified sampling in presence of non-response for fixed cost as well as for fixed precision of the estimate. Khan et al. (2008) suggested a technique for the problem of determining the optimum allocation and the optimum sizes of subsamples to various strata in multivariate stratified sampling in presence of non-response which is formulated as a nonlinear programming problem (NLPP). Varshney et al. (2011) formulated the multivariate stratified random sampling in the presence of non-response as a multi-objective integer nonlinear programming Problem (MINLPP) and a solution procedure is developed using lexicographic goal programming technique to determine the compromise allocation. Haseen et al. (2014) discussed the random cost and random variances in multivariate stratified sampling in presence of non-response and solved the formulated multi-objective nonlinear programming problem (MONLPP) using goal programming, fuzzy programming and D_1 distance method. Fatima and Ahsan (2012) addressed the problem of optimum allocation in stratified sampling under randomized response model as an all integer nonlinear programming problem (AINLPP) in the presence of non-response. Raghav et al. (2012) discussed the various multi-objective optimization techniques in the multivariate stratified sample surveys in case of non-response. Haseen et al. (2012) discussed stochastic multiobjective stratified sampling in presence of non-response.

Geometric programming (GP) is a smooth, systematic and an effective non-linear programming method used for solving problems of sample surveys and engineering design that takes the form of convex programming. The convex programming problems occurring in GP are generally represented by an exponential or power function. Duffin and Zener has done the work in the field of engineering design problems in the early 1960s which was further extended by Duffin et al. (1967). Engineering design problems were also solved by Liu (2008) and Shaojian et al. (2008) with the help of GP. Ojha and Biswal (2010) have worked on posynomial GPPs with multiple parameters. Ojha and Das (2010) have done work on multi-objective GPP with cost coefficients as continuous function with weighted mean. Islam and Roy (2005) discussed the modified GPP and its applications. Dupačová (2010) worked in the field of stochastic geometric programming with an application. Shafiullah et al. (2014) discussed fuzzy geometric programming in multivariate stratified sampling in presence of non-response with quadratic cost.

Davis and Rudolf (1987) applied GP to optimal allocation of integrated samples in quality control. Ahmed and Bonham (1987) applied GP to optimum allocation problems in multivariate double sampling. Maqbool et al. (2011) has discussed the GP approach to find the optimum allocations in multivariate two-stage sampling design. Shafiullah et al. (2013) has worked on GP approach for finding optimum sample sizes in three-stage sample surveys.

In this chapter we have utilized and suggested the GP approach in multivariate stratified sample surveys in case of non-response. The multivariate stratified sample survey in case of non-response has been formulated and solved in two phases. In first phase the multivariate stratified sample surveys in case of non-response has been formulated as GPP and the solution is obtained. The obtained solution is the dual solution of the formulated GPP. In second phase with the help of dual solutions of formulated GPP and primal-dual relationship theorem the optimum allocation of sample sizes of respondents and non-respondents are obtained.

2 Formulation of the problem

In stratified sampling the population of N units is first divided into L non-overlapping subpopulation called strata, of sizes $N_1, N_2, \ldots, N_h, \ldots, N_L$ with $\sum_{h=1}^{L} N_h = N$ and the respective sample sizes within strata are denoted by $n_1, n_2, \ldots, n_h, \ldots, n_L$ with $\sum_{h=1}^{L} n_h = n$.

Let for the h^{th} stratum :

N_h : denote the stratum size,

$\bar{Y}_h$: stratum mean,

S_h^2 : stratum variance,

$W_h = \frac{N_h}{N}$: stratum weight,

N_h : be the sizes of the respondents,

$N_{h2} = N_h - N_{h1}$: be the sizes of non-respondents groups,

$n_h, h = 1, 2 \ldots, L$: units are drawn from the h^{th} stratum.

Further, let out of n_h, n_{h1} units belong to the respondents group.

$n_{h2} = n_h - n_{h1}$: units belong to the non-respondents group,

$n = \sum_{h=1}^{L} n_h$: the total sample size.

A more careful second attempt is made to obtain information on a random subsample of size r_h out of n_{h2} non-respondents for the representation of the non-respondents group from the sample. $r_h = \frac{n_{h2}}{k_h}; h = 1, \ldots, L$ subsamples of sizes at the second attempt to be drawn from n_{h2} non-respondents group of the h^{th} stratum, where $k_h \geq 1$ and $\frac{1}{k_h}$ denote the sampling fraction among non-respondents. Since N_{h1} and N_{h2} are random variables hence their unbiased estimates are given as $\hat{N}_{h1} = \frac{n_{h1} N_h}{n_h}$: unbiased estimates of the respondents group.

$\hat{N}_{h2} = \frac{n_{h2} N_h}{n_h}$: an unbiased estimate of the non-respondents group.

V^* : the upper limits on the variances of each stratum.

$\bar{y}_{jh1}; j = 1, \ldots, p$: denote the sample means of j^{th} characteristic measured on the n_{h1} respondents at the first attempt.

$\bar{y}_{jh2(r_h)}; j = 1, \ldots, p$: denote the r_h sub sampled units from non-respondents at the second attempt.

Using the estimator of Hansen and Hurwitz (1946), the stratum mean $\bar{Y}_{jh}$ of the h^{th} stratum of j^{th} characteristic in the h^{th} stratum may be estimated by

$$y_{j\bar{h}(w)} = \frac{n_{h1}\bar{y}_{jh1} + n_{h2}\bar{y}_{jh2(r_h)}}{n_h} \tag{1}$$

It can be seen that $\bar{y}_{jh(w)}$ is an unbiased estimate of the stratum mean $\bar{Y}_{jh}$ for j^{th} characteristic with a variance.

$$v\big(\bar{y}_{jh(w)}\big) = \left(\frac{1}{n_h} - \frac{1}{N_h}\right) S_{jh}^2 - \frac{W_{h2} S_{jh2}^2}{n_h} + \frac{W_{h2}^2 S_{jh2}^2}{r_h} \tag{2}$$

where S_{jh}^2 is the stratum variance of j^{th} characteristic in the h^{th} stratum; $j = 1, 2, \ldots, p; h = 1, 2, \ldots, L$ given as:

$$S_{jh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} \left(y_{jh1} - \bar{Y}_{jh}\right)^2$$

where y_{jh1} denote the value of the i^{th} unit of the h^{th} stratum for j^{th} characteristic.

$\bar{Y}_{jh} = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{jhi}$: the stratum mean of y_{jhi}.

S^2_{jh2} : denotes the stratum variance of the j^{th} characteristic in the h^{th} stratum among non-respondents, given by:

$$S^2_{jh2} = \frac{1}{\hat{N}_{h2} - 1}\sum_{i=1}^{\hat{N}_{h2}} \left(y_{jhi} - \bar{Y}_{jh2}\right)^2,$$

$\bar{Y}_{jh2} = \frac{1}{\hat{N}_{h2}}\sum_{i=1}^{\hat{N}_{h2}} y_{jhi}$ among non-respondents.

$W_{h2} = \frac{N_{h2}}{N_h}$ is stratum weight of non-respondents in h^{th} stratum.

If the true values of S^2_{jh} and S^2_{jh2} are not known they can be estimated through a preliminary sample or the value of some previous occasion, if available, may be used.

Furthermore, the variance of $\bar{y}_{j(w)} = \sum_{h=1}^{L} W\bar{y}_{jh(w)}$, (ignoring *fpc*) is given as:

$$\begin{aligned} V(\bar{y}_{j(w)}) &= \sum_{h=1}^{L} W_h^2 v(\bar{y}_{jh(w)}) \\ &= \sum_{h=1}^{L} \frac{W_h^2\left(S^2_{jh} - W_{h2}S^2_{jh2}\right)}{n_h} + \frac{W_h^2 W_{h2}^2 S^2_{jh2}}{r_h} \end{aligned} \tag{3}$$

where $\bar{y}_{j(w)}$ is an unbiased estimate of the overall population mean $\bar{Y}_i$ of the j^{th} characteristic, $W_h = \frac{N_h}{N}$ is the stratum weight and $v\left(\bar{y}_{jh(w)}\right)$ is as given in Eq. 2.
Assuming a linear cost function the total cost C of the sample survey may be given as:

$$C = \sum_{h=1}^{L} c_{h0}n_h + \sum_{h=1}^{L} c_{h1}n_{h1} + \sum_{h=1}^{L} c_{h2}n_{h2}$$

where c_{h0} is the per unit cost of making the first attempt,

$c_{h1} = \sum_{j=1}^{p} c_{jh1}$ is the per unit cost for processing the results of all the p characteristics on the n_{h1} selected units from respondents group in the h^{th} stratum in the first attempt and

$c_{h2} = \sum_{j=1}^{p} c_{jh2}$ is the per unit cost for measuring and processing the results of all the p characteristics on the r_h units selected from the non-respondents group in the h^{th} stratum in the second attempt.

Also, c_{jh1} and c_{jh2} are per unit costs of measuring the j^{th} characteristic in first and second attempts respectively. As n_{h1} is not known until the first attempt has been made, the quantity $W_{h1}n_h$ may be used as its expected value. The total expected cost $\hat{C}$ of the survey may be given as:

$$\hat{C} = \sum_{h=1}^{L}(c_{h0} + c_{h1}W_{h1})n_h + \sum_{h=1}^{L} c_{h2}r_h. \tag{4}$$

The problem therefore reduces to find the optimal values of sample sizes of respondents n_1, n_2, n_3, n_4 and non-respondents r_1, r_2, r_3, r_4 which may be expressed as:

$$\left.\begin{array}{ll}
\text{Min } \hat{C} = & \sum\limits_{h=1}^{L}(c_{h0} + c_{h1}W_{h1})n_h + \sum\limits_{h=1}^{L} c_{h2}r_h \\
\text{subject to} & \sum\limits_{h=1}^{L} \frac{W_h^2\left(S_{jh}^2 - W_{h2}S_{jh2}^2\right)}{n_h} + \frac{W_h^2 W_{h2}^2 S_{jh2}^2}{r_h} \leq V^* \\
& n_h, r_h \geq 0. \\
& n_h, r_h \text{ are integers } h = 1, 2, \ldots, L.
\end{array}\right\} \tag{5}$$

3 GP approach in Case of Non-Response in Sample Surveys

In geometric programming technique posynomial functions are minimized subject to several constraints. Posynomial functions can be defined as polynomials in several variables with positive coefficients in all terms and the power to which the variables are raised can be any real number. In sample surveys, the cost function and the variance functions are to be considered in the form of posynomials. Geometric programming always transforms the primal problem of minimizing a "posynomial" subject to "posynomial" constraints to a dual problem of maximizing a function of the weights on each constraint. Generally, constraints are less than strata, so the transformation simplifies the procedure.

The mathematical formulation of problem (5) with the help of given information can be expressed in equation (6) as:

$$\left.\begin{array}{lll}
\text{Min} & f_0(n,r) = \sum\limits_{h=1}^{L} C_h n_h + \sum\limits_{h=1}^{L} C'_h r_h & (i) \\
\text{subject to} & f_i(n,r) = \sum\limits_{h=1}^{L} \frac{a_{1j}}{n_h} + \sum\limits_{h=1}^{L} \frac{a_{2j}}{r_h} \leq V_{0j} & (ii) \\
& n_h, r_h \geq 0. & \\
\text{and} & n_h, r_h \text{ are integers } h = 1, 2, \ldots, L &
\end{array}\right\} \tag{6}$$

where $C_1 = (c_{h0} + c_{h1}W_{h1}), C_2 = c_{h2}, a_{1j} = W_h^2(S_{jh}^2 - W_{h2}S_{jh2}^2)$ and $a_{2j} = W_h^2 W_{h2}^2 S_{jh2}^2$ and V_{oj} are the upper limits on the variances of each stratum.

The following vectors can be found : $(n_1, n_2, n_3, n_4$ and $r_1, r_2, r_3, r_4)$.

The restrictions $n_h \geq 0$ and $r_h \geq 0$ is obvious because the negative values of n_h and r_h are of no practical use.

In the above equations we have noticed that the objective function 6(i) is linear and the constraints 6(ii) are nonlinear and the reduced standard GP (Primal) problem can be stated as:

$$\left.\begin{array}{ll} \text{Min} & f_0(n,r) \\ \text{subject to} & f_j(n,r) \leq 1, j = 1, 2, \ldots, p \\ & n_h, r_h \geq 0. \\ \text{and} & n_h, r_h \text{ are integers } h = 1, 2, \ldots, L \end{array}\right\} \tag{7}$$

where $f_0(n,r) = \sum_{h=1}^{L} C_1 n_h + \sum_{h=1}^{L} C_2 r_h$ and $f_j(n,r) = \sum_{h=1}^{L} \frac{a_{1j}}{n_h} + \sum_{h=1}^{L} \frac{a_{2j}}{r_h} \leq V_{0j}$ are in the form of posynomial functions, where j^{th} posynomial function is given as:

$$\begin{gathered} f_j(n,r) = \sum_{i \in j[q]} \xi_i \Big[\prod_{h=1}^{L} n_h^{p_{ih}} + \prod_{h=1}^{L} r_h^{p_{ih}} \Big], \\ \xi_i \geq 0, n_h, r_h \geq 0; j = 1, 2, \ldots, p; \quad q = 1, \ldots, k \end{gathered} \tag{8}$$

where $\xi_i = \frac{a_{ij}}{V_{0j}} = C_i$ are normalized constants in the i^{th} constraints.

The number of posynomial terms in the function can be denoted by $2L$ as n_h and r_h are two different variables in j^{th} posynomial function with $i = 1, 2, \ldots, 2L$ and $h = 1, 2, \ldots, L$. Also, the exponents p_{ij} are real constants. The objective function $f_0(n,r)$ and the constraint function $f_j(n,r)$ for our allocation problem are given respectively in equation 6(i) and 6(ii).

The dual form of GPP which is stated in equation (6) can be given as:

$$\left.\begin{array}{lll} \text{Max} & v(w) = \left[\prod_{j=0}^{p} \prod_{i \in [j]} \left(\frac{\xi_{ij}}{w_i} \right)^{w_i} \right] \prod_{j=1}^{p} \left(\sum_{i \in [j]} w_i \right)^{\sum_{i \in [j]} w_i} & (i) \\ \text{subject to} & \sum_{i \in [0]} w_i = 1 & (ii) \\ & \sum_{j=0}^{p} \sum_{i \in [j]} p_{ij} w_i = 0 & (iii) \\ & w_i \geq 0, j = 0, \ldots, p & (iv) \end{array}\right\} \tag{9}$$

where w_i's are dual variables.

The stepwise formulation of the problem (9) is as follows:

Step 1: The objective function takes the form:

$$\begin{aligned} C_0(x^*) &= \left(\frac{\text{coeff. of 1st term}}{w_1}\right)^{w_1} \times \ldots \times \left(\frac{\text{coeff. of last term}}{w_L}\right)^{w_L} \\ &\times (\textstyle\sum w\text{'s in 1st constraint})^{\sum(w\text{'s in 1st constraint})} \times \ldots \\ &\times (\textstyle\sum w\text{'s in last constraint})^{\sum(w\text{'s in last constraint})} \end{aligned}$$

The objective function (i.e. cost function) for our problem is:

$$\left[\prod_{j=0}^{p} \prod_{i\in[j]} \left(\frac{\xi_{ij}}{w_i}\right)^{w_i}\right] \prod_{j=1}^{p} \left(\sum_{i\in[j]} w_i\right)^{\sum\limits_{i\in[j]} w_i}, \quad j = 1, 2, \ldots, p \tag{10}$$

where $\xi_{ij} = \frac{a_{ij}}{V_{oj}} = C_i,\ i = 1, 2.\&\ \ j = 1, 2, \ldots, p$.

Step 2: The equations that can be used for geometric program for the weights are given below:
$\sum\limits_{i\in[0]} w_i$ in the objective function= 1(Normality condition ,see equation 9(ii))
i.e. $w_{01} + w_{02} + \ldots + w_{0h} + \ldots + w_{0L} = 1$

and for each primal variable n_i and r_i given n variables and L terms:

$\sum\limits_{i=1}^{L}$ (exponents on n_i and r_i)$\times$(w_i for each term)=0(Orthogonality condition, see equation 9(iii))

and $w_i \geq 0$ (Positivity condition, see equation 9(iv))
The dual problem (9) has been formulated with the help of above steps and the corresponding solution w^*_{0i} is unique to the dual constraints, it will also maximize the objective function for the dual problem. Next, the solution of the primal problem will be obtained using primal-dual relationship theorem which is given below:

Primal-dual relationship theorem: If w^*_{0i} is a maximizing point for dual problem (9), each minimizing point n_1, n_2, n_3, n_4 and r_1, r_2, r_3, r_4 for primal problem (6) satisfies the system of equations:

$$f_0(n, r) = \begin{cases} w^*_{0i} v(w^*), & i \in J[0], \\ \frac{w_{ij}}{v_L(w^*_{0i})}, & i \in J[L], \end{cases} \tag{11}$$

where L ranges over all positive integers for which $v_L(w^*_{0i}) > 0$.

The optimal values of respondents n^*_h and non-respondents r^*_h can be calculated with the help of the primal-dual relationship theorem (11).

4 Numerical Illustrations:

A numerical example is given to demonstrate the proposed method. The values of S^2_{jh2} and S^2_{jh} are practically unknown. Their values on some previous occasion may be used. It is assumed that the relative values of the stratum variances among the non-respondents at the second attempt to the corresponding over all stratum variances are $\frac{S^2_{jh2}}{S^2_{jh}} = 0.25; h = 1, 2, \ldots, L$ and $j = 1, 2, \ldots, p$. This ratio has been taken as 0.25 in the example for the sake of simplicity. Practically this ratio may vary from stratum to stratum and from characteristic to characteristic.

Example: The data is taken from Khan et al. (2008). Consider a population of size $N = 3850$ divided into four strata. The two characteristics are defined on each unit of the population and the population means are to be estimated. The available information is shown in the given table.

Table 1: Data for four Strata and two characteristics

h	N_h	S^2_{1h}	S^2_{2h}	w_{h1}	w_{h2}	c_{h0}	c_{h1}	c_{h2}
1	1214	4817.72	8121.15	0.70	0.30	1	2	3
2	822	6251.26	7613.52	0.80	0.20	1	3	4
3	1028	3066.16	1456.4	0.75	0.25	1	4	5
4	786	6207.25	6977.72	0.72	0.28	1	5	6

On substituting the values in equation (6) from the table 1, we have obtained the expression (12) given below:

$$\left.\begin{array}{ll}
\text{Min} & C = 2.4n_1 + 3.4n_2 + 4n_3 + 4.6n_4 + 3r_1 + 4r_2 + 5r_3 + 6r_4 \\[2ex]
\text{subject to} & \frac{101.4077}{n_1} + \frac{58.19923}{n_2} + \frac{46.56731}{n_3} + \frac{51.31327}{n_4} \\[2ex]
 & \frac{2.46667}{r_1} + \frac{0.612623}{r_2} + \frac{0.776122}{r_3} + \frac{1.081441}{r_4} \leq 1 \\[2ex]
 & \frac{153.8471}{n_1} + \frac{63.7968}{n_2} + \frac{19.90717}{n_3} + \frac{51.31327}{n_4} \\[2ex]
 & \frac{3.74226}{r_1} + \frac{0.671512}{r_2} + \frac{0.331786}{r_3} + \frac{1.094106}{r_4} \leq 1 \\[2ex]
 & n_h, \geq 0, r_h \geq 0; h = 1, 2, \ldots, L \\[2ex]
\text{and} & n_h,\ r_h \text{ are integers.}
\end{array}\right\} \quad (12)$$

The corresponding dual problem of primal problem expression (12) is:

$$\left.\begin{aligned}
&\max v(w_{0i}^{*}) = ((2.4/w_{01})^{w_{01}}) \times ((3.4/w_{02})^{w_{02}}) \times ((4/w_{03})^{w_{03}}) \times ((4.6/w_{04})^{w_{04}}) \\
&\times ((3/w_{05})^{w_{05}}) \times ((4/w_{06})^{w_{06}}) \times ((5/w_{07})^{w_{07}}) \times ((6/w_{08})^{w_{08}}) \\
&\times ((101.4077)^{w_{11}}) \times ((58.19923)^{w_{12}}) \times ((46.56731)^{w_13}) \times ((51.31327)^{w_{14}}) \\
&\times ((2.46667)^{w_{15}}) \times ((0.612623)^{w_{16}}) \times ((0.776122)^{w_{17}}) \times ((1.081441)^{w_{18}}) \\
&\times ((153.8471)^{w_{21}}) \times ((63.7968)^{w_{22}}) \times ((19.90717)^{w_{23}}) \times ((51.91424)^{w_{24}}) \\
&\times ((3.74226)^{w_{25}}) \times ((0.671512)^{w_{26}}) \times ((0.331786)^{w_{27}}) \times ((1.094106)^{w_{28}}) \qquad (i) \\
&\text{subject to} \\
&w_{01} + w_{01} + w_{03} + w_{04} + w_{05} + w_{06} + w_{07} + w_{08} = 1 \qquad \text{(normality condition)} \quad (ii) \\
&\left.\begin{aligned}
&w_{01} - w_{11} - w_{21} = 0 \\
&w_{02} - w_{12} - w_{22} = 0 \\
&w_{03} - w_{13} - w_{23} = 0 \\
&w_{04} - w_{14} - w_{24} = 0 \\
&w_{05} - w_{15} - w_{25} = 0 \\
&w_{06} - w_{16} - w_{26} = 0 \\
&w_{07} - w_{17} - w_{27} = 0 \\
&w_{08} - w_{18} - w_{28} = 0
\end{aligned}\right\} \qquad \text{(orthogonality condition)} \qquad (iii) \\
&\left.\begin{aligned}
&w_{01}, w_{02}, w_{03}, w_{04}, w_{05}, w_{06}, w_{07}, w_{08} > 0 \\
&w_{11}, w_{12}, w_{13}, w_{14}, w_{15}, w_{16}, w_{17}, w_{18}, \\
&w_{21}, w_{22}, w_{23}, w_{24}, w_{25}, w_{26}, w_{27}, w_{28} \geq 0
\end{aligned}\right\} \qquad \text{(positivity condition).} \qquad (iv)
\end{aligned}\right\} \tag{13}$$

For orthogonality condition defines in expression problem 13(iii) are evaluated with the help of the orthogonality payoff matrix and from this payoff matrix make the orthogonality condition equation which as defines below:

$$\begin{pmatrix} 1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix} \times \begin{pmatrix} w_{01} \\ w_{02} \\ w_{03} \\ w_{04} \\ w_{05} \\ w_{06} \\ w_{07} \\ w_{08} \\ w_{11} \\ w_{12} \\ w_{13} \\ w_{14} \\ w_{15} \\ w_{16} \\ w_{17} \\ w_{18} \\ w_{21} \\ w_{22} \\ w_{23} \\ w_{24} \\ w_{25} \\ w_{26} \\ w_{27} \\ w_{28} \end{pmatrix}$$

$$= \left. \begin{array}{r} w_{01} - w_{11} - w_{21} = 0 \\ w_{02} - w_{13} - w_{22} = 0 \\ w_{03} - w_{13} - w_{23} = 0 \\ w_{04} - w_{14} - w_{24} = 0 \\ w_{05} - w_{15} - w_{25} = 0 \\ w_{06} - w_{16} - w_{26} = 0 \\ w_{07} - w_{17} - w_{27} = 0 \\ w_{08} - w_{18} - w_{28} = 0 \end{array} \right\}.$$

After solving the formulated dual problem (13) using lingo software we obtain the following values of the dual variables which are given as:
$w_{01} = 0.35655525, w_{02} = 0.2094496, w_{03} = 0.1798722, w_{04} = 0.2306042,$
$w_{05} = 0.0108412, w_{06} = 0.002593801, w_{07} = 0.003747338, w_{08} = 0.006339187$
and $v(w^*_{0i}) = 1035.564$

Using the primal dual relationship theorem (11), we have the optimal solution of primal problem: i.e., the optimal sample sizes of respondents and non-respondents are computed as follows:

$$f_0(n, r) = w^*_{0i} v(w^*) \tag{14}$$

In expression (14), we first keep the r constant and calculate the values of n as:

$f_{01}(n_1, r) = w^*_{01} v(w^*)$
$2.4 \times n_1 = 0.365525 \times 1035.564$
$\Rightarrow n_1 \cong 154$

$f_{02}(n_2, r) = w^*_{02} v(w^*)$
$3.4 \times n_2 = 0.2094496 \times 1035.564$
$\Rightarrow n_1 \cong 64$

$f_{03}(n_3, r) = w^*_{03} v(w^*)$
$4 \times n_3 = 0.1798722 \times 1035.564$
$\Rightarrow n_3 \cong 47$

$f_{04}(n_4, r) = w^*_{04} v(w^*)$
$4.6 \times n_4 = 0.2306042 \times 1035.564$
$\Rightarrow n_4 \cong 52$

Now, from the expression (13), we keep the n constant and calculate the values of r as:

$f_{01}(n, r_1) = w^*_{01} v(w^*)$
$4 \times r_1 = 0.01084122 \times 1035.564$
$\Rightarrow r_1 \cong 4$

$f_{02}(n, r_2) = w^*_{02} v(w^*)$
$4 \times r_2 = 0.002593801 \times 1035.564$
$\Rightarrow r_2 \cong 1$

$f_{03}(n, r_3) = w^*_{03} v(w^*)$
$5 \times r_3 = 0.003747338 \times 1035.564$
$\Rightarrow r_3 \cong 1$

$f_{04}(n, r_4) = w^*_{04} v(w^*)$
$6 \times r_4 = 0.006339187 \times 1035.564$
$\Rightarrow r_4 \cong 1$

The optimal values and the objective function value are given below:
$n^*_1 = 154, n^*_2 = 64, n^*_3 = 47$ and $n^*_4 = 52$;
$r^*_1 = 4, r^*_2 = 1, r^*_3 = 1$ and $r^*_4 = 1$ and the optimal value of the objective in primal problem is 1037.4

5 Conclusions

In this paper the multivariate stratified sample problem in case of non-response is formulated as a geometric programming problem and the allocation of sample sizes of respondents and non-respondents are obtained. The problem of multivariate stratified sample in case of non-response is solved in two phases. In the first phase given problem is formulated as GPP and the solution is obtained. The obtained solution is the dual solution of the formulated GPP. In second phase with the help of dual solutions of formulated GPP and primal-dual relationship theorem the optimum allocation of sample sizes of respondents and non-respondents are obtained. Many authors have discussed the same problem with different methods and obtained the allocations but in this paper a comprehensive study of GP approach in multivariate stratified sample surveys in case of non-response is provided with suggestion.

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